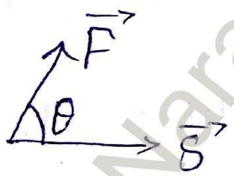
Work, Energy & power

Work:- If an object get displaced by a displacement $\vec{S}$ under influence of a force $\vec{F}$ then work is said to be done by the force.

Mathematically

$$W = \vec{F} \cdot \vec{S}$$



$$\boxed{W = FS\cos\theta}$$

→ Work is a scalar quantity.

→ unit → N-m (in → Joule)

→ $[W] = ML^2T^{-2}$

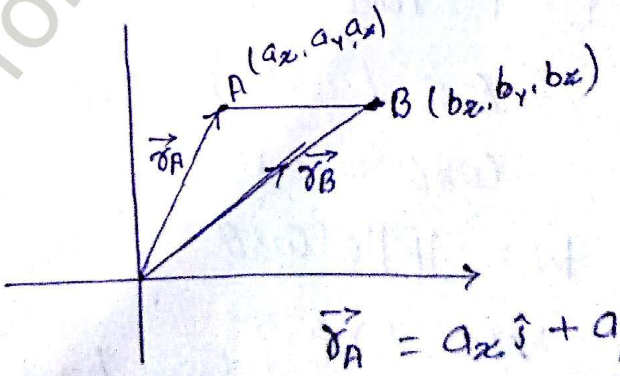
Let $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and

$\vec{S} = x\hat{i} + y\hat{j} + z\hat{k}$

$W = \vec{F} \cdot \vec{S}$

$= (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})(x\hat{i} + y\hat{j} + z\hat{k})$

$$\boxed{W = F_x \cdot x + F_y \cdot y + F_z \cdot z}$$



$\vec{r}_A = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$\vec{r}_B = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$

$$\therefore \vec{AB} = \vec{r}_B - \vec{r}_A$$

$$= (b_x\hat{i} + b_y\hat{j} + b_z\hat{k}) - (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})$$

$$\vec{AB} = [(b_x - a_x)\hat{i} + (b_y - b_x) + (b_z - a_z)\hat{k}]$$

If $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$$W = \vec{F}\cdot\vec{S} = \vec{F}\cdot\vec{AB}$$

$$\boxed{W = F_x(b_x - a_x) + F_y(b_y - a_y) + F_z(b_z - a_z)}$$

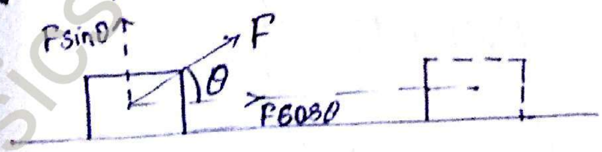
Calculation of work done:-

1) Work done by a constant force.
2) Work done by a variable force.
3) Work done by using Force-displacement curve.

(1) Work done by a constant force.

$$W = \vec{F}\cdot\vec{S} = FS\cos\theta$$

Case-1.



$$\theta \in (0, \pi/2)$$

$$\cos\theta = +ve.$$

$$W = |\vec{F}||\vec{S}|\cos\theta$$

$$W = +ve$$

Work is said to be done by the force.

$$W = \{|\vec{F}|\cos\theta\}|\vec{S}|$$

= Displacement × Component of force along the direction of displacement.

Case :- 2.

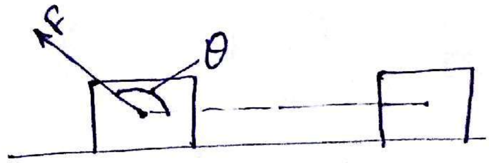
$\theta = 90°$

$\cos\theta = 0$

$$\boxed{W = 0}$$

No work is done by the force F

Case - 3.



$\theta \in (\pi/2, \pi)$

$\cos\theta = -ve$

$W = FS\cos\theta = -ve.$

Work is done against this force.

$$W = F \cdot S\cos\theta \begin{cases} \theta \in (0, \pi/2) & W = +ve \\ \theta \in (\pi/2) & W = 0 \\ \theta \in (\pi/2, \pi) & W = -ve. \end{cases}$$

Work done by a variable force:-

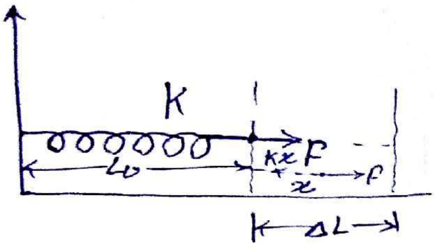
If force acting on an object is given by ~~to~~ its function of displacement than work done by the force will be calculated as.

$$\left[W = \int_{r_i}^{r_f} \vec{F}_{(\vec{r})} \cdot d\vec{r} \right]$$

r_i = It is position vector crosponding to initial position.

r_f = It is position vector corresponding to final position.

Ex:-



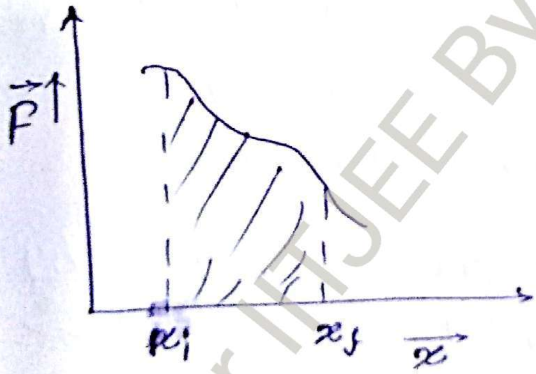
$$d\vec{W} = \vec{F}\cdot dx$$
$$= Kx\cdot dx$$

Integrating both side.

$$W = K\int_0^{\Delta L} x\,dx$$

$$\boxed{W = \frac{1}{2}K\Delta L^2}$$

$$\boxed{W = \int_{x_i}^{x_f} F\cdot dx}$$



→ If area (A) is comes out +ve, than
$W = +ve$

→ If area(A) is comes out −ve than
$W = -ve$

Energy

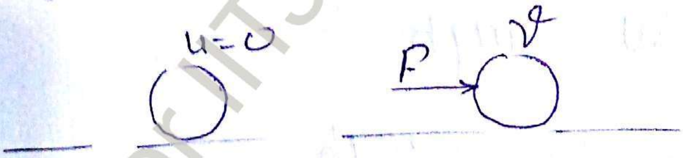
Energy defined as capacity to do work.

Energy

- Mechanical Energy
  - Kinetic Energy
  - potential Energy.
- Non mechanical Energy.
  - → thermal Energy
  - → Light Energy
  - → Electrical Energy
  - → wind Energy.
  - --- etc.

Kinetic Energy :-

It is defined as Energy possesed by an object by virtu of its motion.



KE = work done by F

$$dw = F \cdot ds.$$

$$= ma \cdot ds$$

$$= mv \cdot \frac{dv}{ds} \cdot ds$$

$$W = \int_{u=0}^{v} mv \cdot dv$$

$$W = \frac{1}{2} mv^2$$

SI unit is Joule.

$$\boxed{\therefore \; K \cdot E = \frac{1}{2} mv^2}$$

Potential Energy

It is defined as Energy possesed by an object by virtue of its position or Configration.

→ Gravitational potential Energy.

→ Elastic potential Energy.

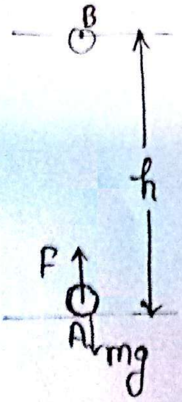
→ Electrostatic potential Energy.

$$dU = -dW$$

$$= \vec{F}\cdot\vec{dr}$$

$$\boxed{\vec{F} = \frac{-dU}{\vec{dr}}}$$ U = potential Energy.

Gravitational potential Energy.



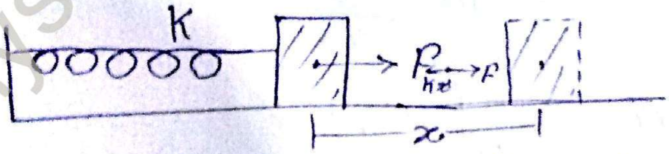
$$W = F\cdot h$$

$$\Delta U = mgh$$

$$U_A - U_B = -mgh$$

$$\boxed{U_B = mgh}$$

Elastic potential Energy:-



U = work done by F

$$= \int_0^x F\cdot dx = \int_0^x Kx\,dx = \frac{1}{2}Kx^2$$

$$\boxed{U = \frac{1}{2}Kx^2}$$ Elastic potential energy of the spring.

Power

It is defined as rate of doing a work.

$$P = \frac{dW}{dt}$$

$$(\text{Avg. power})\ P = \frac{W}{\Delta t} = \frac{\text{total work done}}{\text{time taken}}$$

$$P = \frac{dW}{dt}$$

$$W = F \cdot S$$

$$\frac{dW}{dt} = \frac{d(F \cdot S)}{dt}$$

$$= F \cdot \frac{dS}{dt} \quad (\text{if } F \text{ is Constant})$$

$$\boxed{P = F \cdot V}$$

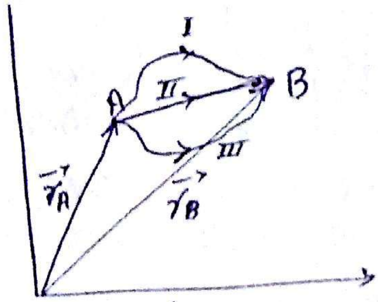
→ SI unit → J/sec.

1 J/sec = Watt

→ 1 hp = 746 watt

Classification of force:-

$$W = \vec{F}\cdot d\vec{s}$$



$W_I = W_{II} = W_{III}$	$W_I \neq W_{II} \neq W_{III}$
Conservative force	**Non Conservative force**
1. path independent.	1. path dependent.
2. If work done through a closed path is zero	2. If work done through a closed path not equal to zero.
$W = \oint \vec{F}\cdot d\vec{s} = 0$	$W = \oint \vec{F}\cdot d\vec{s} \neq 0$
Ex:- gravitational force. Electrostatic force.	Ex- Friction, viscuse force

Work Energy theorem

work done by all forces on an object is always equals to change in kinetic energy of the object.

$$W = \Delta KE$$
$$= KE_f - KE_i$$

$$\boxed{W = \frac{1}{2}m(v_f^2 - v_i^2)}$$

If $v_f > v_i \longrightarrow W = +ve$

If $v_f < v_i \longrightarrow W = -ve$.

Law of Conservation of energy

From definition of P.E

$$U_f - U_i = -W \quad \left[\because F = \frac{-dU}{dr}\right]$$

$$U_f - U_i = -[K.E_f - KE_i] \quad \left[\because \text{from work energy theorem } W = \Delta KE = KE_f - KE_i\right]$$

$$U_f - U_i = -KE_f + KE_i$$

$$U_f + KE_f = U_i + KE_i$$

$$\boxed{E_f = E_i}$$ This is known as Law of Conservation of Energy.

If

$$W = \Delta KE$$

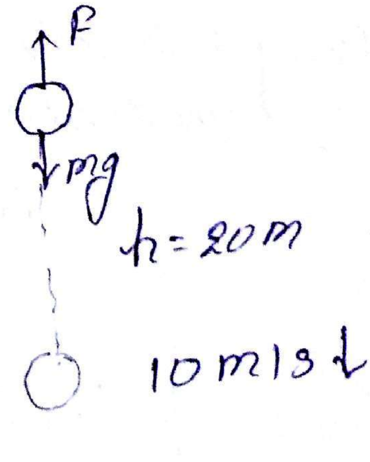
$$W_{conse} + W_{nonconser} + W_{Ex} = \Delta KE$$

$$\Rightarrow -(U_f - U_i) + W_{NC} + W_{Ex} = \Delta KE$$

$$\Rightarrow \boxed{W_{NC} + W_{Ex} = \Delta KE + \Delta U}$$

Q. An object of mass 5 kg is released from rest and attain a speed of 10 m/s after falling through a height of 20 m. Find the work done by air Resistance.

Soln From work energy theorem.



$$W = \Delta KE$$

$$\Rightarrow W_{mg} + W_{air} = \Delta KE$$

$$\Rightarrow mgh + W_A = KE_f - KE_i$$

$$\Rightarrow mgh + W_{air} = \frac{1}{2}mv^2 - 0$$

$$W_{Air} = \frac{1}{2}mv^2 - mgh$$

$$= \frac{1}{2} \times 5 \times (10)^2 - 5 \times 10 \times 20$$

$$= 5 \times 50 - 10 \times 100$$

$$= 250 - 1000$$

$$= -750\ J \quad \underline{\text{Ans}}$$

Q. The displacement of a particle moving in on dimension is given by

$$t = \sqrt{x} + 3 \qquad t \to \text{sec.}, \quad x \to \text{meter}$$

then find the work done in 1st 6 sec. of the motion of the object.

Soln

$$t = \sqrt{x} + 3$$

$$\Rightarrow (t-3)^2 = x$$

$$\Rightarrow x = (t-3)^2$$

$$\frac{dx}{dt} = 2(t-3)$$

$\therefore$ speed of particle

$$v_{t=0} = -6\ m/s. \quad \therefore (KE)_{t=0} = \frac{1}{2}m(-6)^2$$

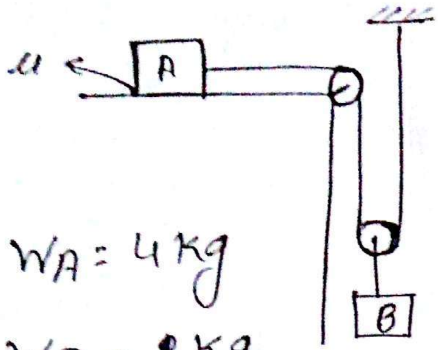
$$v_{t=6} = 6\ m/s. \quad \therefore (KE)_{t=6} = \frac{1}{2}m(6)^2$$

$$(KE)_{t=0} = (KE)_{t=6}$$

$$\therefore \Delta KE = 0$$

$$\boxed{\therefore W = 0}$$

8.



$W_A = 4\,kg$

$W_B = 2\,kg$

After system is released from rest mass B acquires a velocity of 0.3 m/s after falling through a height of 1 m find the value of co-efficient of friction μ.

Soln

$$V_B = \frac{1}{2} V_A$$

$$2V_B = V_A = 0.6\,m/s.$$

$$W_{all} = \Delta KE$$

$$W_{fri} + W_{gra} = \Delta KE$$

$$\Rightarrow -\mu m_A g \times 2 + m_B g \times 1 = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$$

$$\Rightarrow -\mu \times 4 \times 10 \times 2 + 2 \times 10 \times 1 = \frac{1}{2} 4 \times (0.6)^2 + \frac{1}{2} \times 1 \times (0.3)^2$$

$$\Rightarrow -80\mu + 10 = 2 \times 0.36 + \frac{1}{2} \cdot 0.09$$

$$\Rightarrow -80\mu = 10 - 0.72 - 0.045$$

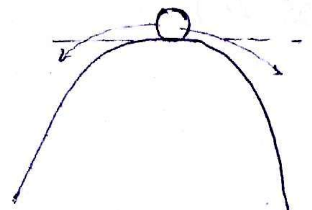
$$\boxed{\mu = 0.115}\ \text{Ans}$$

Different type of Equilibrium

$$F = 0 \qquad \left[\because F = -\frac{dU}{dr}\right]$$

$$\frac{dU}{dr} = 0$$

case-I



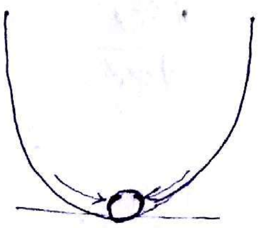
unstable equilibrium

$$\frac{dU}{dr} = 0$$

$P.E = U = max^m$

$$\frac{d^2U}{dr^2} < 0$$

case-II



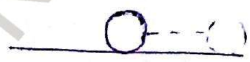
stable equilibrium.

$$\frac{dU}{dr} = 0$$

$PE = U \rightarrow min^m$

$$\frac{d^2U}{dr^2} > 0$$

case-III



Nutral equi.

$$\frac{dU}{dr} = 0$$

$P.E = U \rightarrow$ neither max^m or nor min^m

$$\frac{d^2U}{dr^2} = 0$$

<u>Q.</u> If potential energy of an object moving along X-axis is given as $U = ax^2 - bx$ where a and b are +ve Constants then find the position and nature of equilibrium if any.

<u>Soln:-</u>

$$F = 0$$

$$\frac{dU}{dx} = 0$$

$$\Rightarrow \frac{d(ax^2 - bx)}{dx} = 0$$

$$\Rightarrow 2ax - b = 0$$

$$x = b/2a$$

$$\frac{d^2U}{dx^2} = \frac{d}{dx^2}(ax^2 - bx)$$

$$\Rightarrow \frac{dU^2}{dx^2} = 2a$$

$$\frac{dU^2}{dx^2} > 0$$

$\therefore x = \frac{b}{2a}$ will be a point of minima and hence stable equi.

Q. potential Energy of a particle moving along x-axis is given by.

$$U = \frac{x^3}{3} - 4x + 6$$ x is in meter an U is in Jule.

Find the all equilibrium position of particle and thus type.

Soln

$F = 0$

$\Rightarrow \frac{dU}{dx} = 0$

$\Rightarrow \frac{d\left(\frac{x^3}{3} - 4x + 6\right)}{dx}$

$\Rightarrow \frac{3x^2}{3} - 4 = 0$

$x = \pm 2$

$\frac{d^2u}{dx^2} = \frac{d}{dx}(x^2 - 4)$

$= 2x$

$\left(\frac{d^2u}{dx^2}\right)_{x=2} = 4 = +ve$

$\left(\frac{d^2u}{dx^2}\right)_{x=-2} = -4 = -ve$

$(x = +2)$ mima, stable equi

$(x = -2)$ maxima, unstable equi-

Q.

A particle of mass 1kg is moving with an accl^n of $4\,m/s^2$ starting from rest at $t = 0$

(i) Find the avg. power of the force in time $t = 0$ to $t = 2$

(b) Find the instantenous power of the force at $t = 4$ sec.

Soln (i) $(P)_{avg} = \frac{W}{t}$

$= \frac{\Delta E}{t}$

$= \frac{\frac{1}{2}mv^2 - 0}{2}$

$= \frac{1}{4}mv^2$

$= \frac{1}{4}m(at)^2$

$= \frac{1}{4}ma^2t^2$

$= \frac{1}{4} \times 1 \times 4^2 \times 2^2$

$= 16\,J/s = 16$ watt

Ans

$$S = \frac{1}{2} a t^2$$

$$= \frac{1}{\cancel{2}}\left(\frac{\cancel{20}^{10}}{\cancel{8}}\right) \times 8^2$$

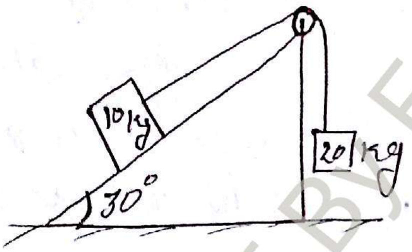
$$\boxed{S = 80\ m}$$

$$W = F \cdot S.$$

$$= T \cdot S \cos 0^\circ$$

$$= \frac{300}{8} \times \cancel{80}^{10} \times 1 = 3000\ J.$$

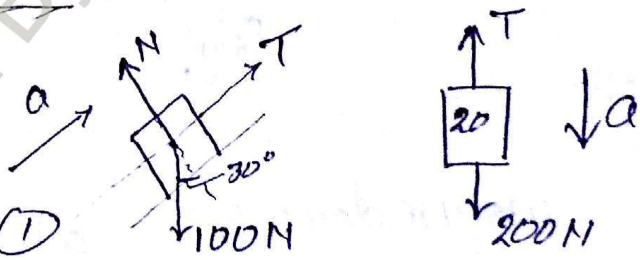
Q. Find the work done by gravitational force on 10 kg block in 2 sec after system is released from rest.



F.B.D

Soln

Eqn of motion of 20 kg



$$200 - T = 20a \quad \text{—— (I)}$$

eqn of motion of 10 kg

$$T - 100 \sin 30 = 10a \quad \text{—— (II)}$$

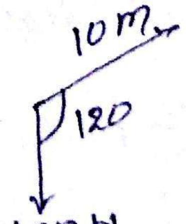
adding (I) and (II)

$$150 = 30a$$

$$\Rightarrow a = \frac{150}{30} = 5\ m/s^2$$

$$S = \frac{1}{2} a t^2$$

$$= \frac{1}{2} \cdot 5(2)^2 = 10\ m$$



$$W = 100 \cdot 10 \cdot \cos 120^\circ$$

$$= -100 \times \cancel{10}^{5} \times \frac{1}{\cancel{2}}$$

$$\boxed{W = -500\ J}$$

(2.)

$$P = \frac{dw}{dt}$$

$$P = \frac{d(F\cdot s)}{dt}$$

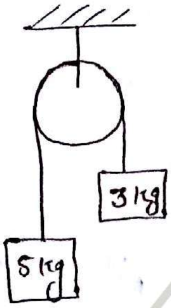
$$= \frac{d}{dt}(ma\cdot s)$$

$$= ma\left\{\frac{ds}{dt}\right\}_{t=4}$$

$$= ma\cdot a\cdot t = ma^2 t$$

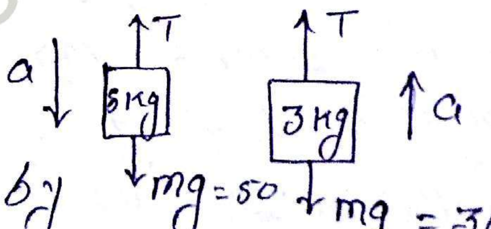
$$= 1\times 4^2\times 4 = 64\ J/s = 64\ \text{watt.}$$

Q. Find the work done by a string in 8 sec. after system is set in to motion.
& find the work done by the string on 3 kg block in 8 sec. after sys. is released.



Soln

work done by string on 3 kg block
= −{work done by string on 5 kg block}



Total work done by string = 0

& for 3 kg block

$T - 30 = 3a$ —— (I)

for 5 kg block

$50 - T = 5a$ —— (II)

adding eqn (I) and (II)

$8a = 20$

$a = 20/8\ m/s^2$

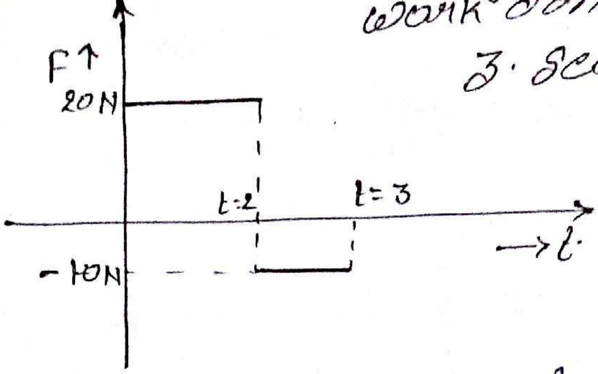
$T = 30 + 3a$

$= 30 + 3\left(\frac{20}{8}\right)$

$= \frac{240+60}{8}$

$= \frac{300}{8}\ N$

Q. An object of mass 5 kg is acted upon by one force as shown in fig. Find the total work done by the force in 3 sec.



Soln

Work done in $t = 0$ to $t = 2$ sec.

$F = 20\,N, \; m = 5\,kg$

$\therefore \; a = F/m = \frac{20}{5} = 4\,m/s^2$

$s = \frac{1}{2}at^2$

$= \frac{1}{2} \cdot 4 \times 2^2 = 8\,m$

$W = 20 \times 8 = 160\,J.$

velocity at 2 sec

$v = at$

$= 4 \times 2 = 8\,m/s.$

Work done in $t = 2$ to $t = 3$ sec.

$F = -10\,N, \; m = 5\,kg$

$a = F/m = \frac{10}{5} = -2\,m/s^2$

$s = \frac{1}{2}at^2 + ut.$

$= \frac{1}{2}(-2)1^2 + 4t$

$= \frac{1}{2} \times -2 + 8 \times 1$

$\Rightarrow 7\,m$

$W = -10 \times 7 = -70\,J.$

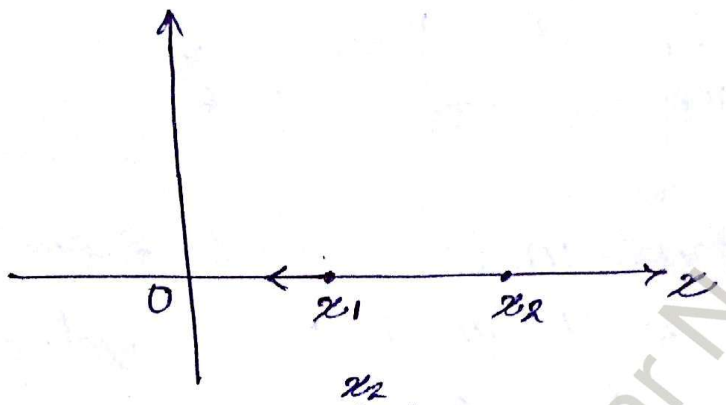
Total workdone

$W = 160 - 70$

$= 90\,J.$ **Ans**

Q. An object is attracted towards origin with a force $F_x = -\frac{K}{x^2}$ find the workdone by the force when the object is moved along x-axis from x_1 to x_2 $(x_2 > x_1)$

Soln



$$W = \int_{x_1}^{x_2} \vec{F} \cdot d\vec{x}$$

$$= \int_{x_1}^{x_2} -F \cdot dx$$

$$= -\int_{x_1}^{x_2} K/x^2 \, dx = -K \int_{x_1}^{x_2} \frac{1}{x^2} dx$$

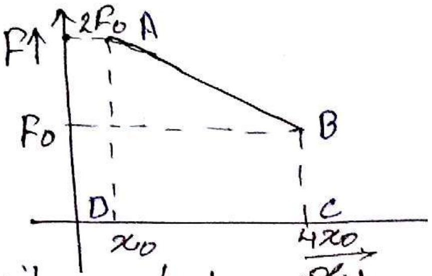
$$= -K \left[-\frac{1}{x} \right]_{x_1}^{x_2}$$

$$W = -K \left[\frac{-1}{x_2} + \frac{1}{x_1} \right]$$

$$= -K \left[\frac{-x_1 + x_2}{x_1 x_2} \right]$$

$$= -K \left[\frac{x_2 - x_1}{x_1 x_2} \right] \quad \text{Ans}$$

Q. A particle of mass m is moving along a straight line under influence of force which varies with its position as shown.



If the velocity of particle at position x_0 is $\sqrt{\frac{2F_0x_0}{m}}$ then find the velocity of particle at position $4x_0$

Soln :-

Work done by the force will be area of ABCD

$$W = \frac{1}{2}(2F_0+F_0)\times 3x_0$$

$$= \frac{1}{2}3F_0\times 3x_0$$

$$\boxed{W = \frac{9F_0x_0}{2}} \quad \text{--- (i)}$$

From work energy theorem

$$W = \Delta KE$$

$$= KE_f - KE_i$$

$$\Rightarrow \frac{9F_0x_0}{2} = KE_f - \frac{1}{2}mv_i^2$$

$$\Rightarrow \frac{9F_0x_0}{2} = KE_f - \frac{1}{2}\cancel{m}\cdot\left(\frac{\cancel{2}F_0x_0}{\cancel{m}}\right)$$

$$\therefore KE_f = F_0x_0\left(\frac{9}{2}+1\right) = \frac{11}{2}F_0x_0$$

$$\frac{1}{\cancel{2}}mv_f^2 = \frac{11}{\cancel{2}}F_0x_0$$

$$\boxed{v_f = \sqrt{\frac{11F_0x_0}{m}}} \quad \underline{\text{Ans}}$$

Q. A particle is moving under influence of a force given by $F = \pi \sin(\pi x)$. Find the work done by the force in slowly moving it $x=0$ to $x = 0.5\,m$.

Soln :- In case of work done by variable force.

$$W = \int_{x_i}^{x_f} F \cdot dx$$

$$\Rightarrow W = \int_0^{x=0.5} \pi \sin(\pi x)\,dx$$

$$= \pi \int_0^{0.5} \sin(\pi x)\,dx$$

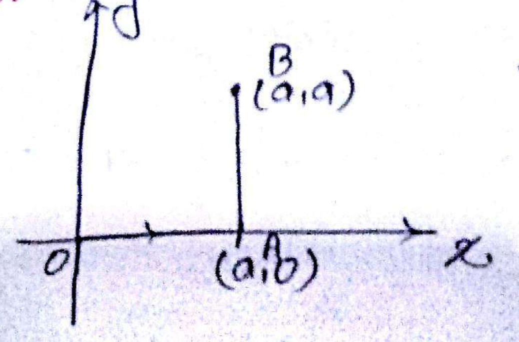
$$= \pi \left[-\frac{\cos(\pi x)}{\pi} \right]_0^{0.5}$$

$$= \pi \left[-\frac{\cos \pi/2}{\pi} + \frac{\cos 0}{\pi} \right]$$

$$= \cancel{\pi} \left[0 + \frac{1}{\cancel{\pi}} \right] = 1\,J \quad \underline{\text{Ans}}$$

Q. A force $\vec{F} = -k(y\hat{i} + x\hat{j})$ acts on a particle moving xy plane. If the particle starts from origin and taken to a point $(a,0)$ along x axis and taken to a point (a,a) along a line parallel to y axis than calculate the total work done by the force.

Soln



$$W = W_{OA} + W_{AB}$$

$\overrightarrow{OA}$

$\vec{F} = -k(x)\hat{j}$

$\overrightarrow{OA} = a\hat{i}$

$W = \vec{F}\cdot\overrightarrow{OA}$

$\boxed{W_{OA} = 0}$

AB

$\vec{F} = -k(y\hat{i} + a\hat{j})$

$\overrightarrow{AB} = a\hat{j}$

$W_{AB} = -k(y\hat{i}+a\hat{j})\cdot a\hat{j}$

$= -ka^2$

$W = W_{OA} + W_{AB}$

$W = 0 - ka^2$

$\boxed{W = -ka^2}$ Ans